

Title: Reasonable Estimates

Brief Overview:

Estimation is a critical component of number sense. Through the activities and math games in this unit, students will practice identifying appropriate situations for estimation, calculating estimated sums and differences, and evaluating answers for reasonableness. They will have the opportunity to develop flexibility in their math thinking.

NCTM Content Standard/National Science Education Standard:

1. Compute fluently and make reasonable estimates
 - Develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results;
 - Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected methods or tools.

Grade/Level:

Grades 3-4

Duration/Length:

3 days (60 minutes each day)

Student Outcomes:

Students will:

- Determine whether an estimate or an exact answer is appropriate for a given situation, and explain rationale.
- Estimate sums of two or three addends, and explain strategies.
- Estimate differences and evaluate them for reasonableness.

Rationale:

Time spent “teaching” students to estimate, and evaluate estimations, is valuable. These types of activities are critical in students’ development of number sense. They provide students the opportunity to learn and hone a skill that they will use in the real world. Mental computation is an important part of number sense. Mental math requires mathematical dexterity in manipulating numbers so that they are easy to compute. Accommodations can be made for students with special needs, though all children should have ample opportunity to practice estimating,

and evaluate responses. Students who can estimate are able to judge whether their math calculations—even those typed on a calculator—are “reasonable.”

Encourage students to use a variety of estimation strategies including front-end estimation, rounding, and compatible numbers. These three strategies can be applied to the same (addition or subtraction) problems. For example:

$$355 + 57$$

Front end $300 + 50$;

Round to the nearest hundred or ten $400 + 60$ or $360 + 60$;

Compatible numbers $350 + 50$

Assumption:

For this unit, students should be able to add up to three addends through the hundreds place with and without regrouping; subtract three digit numbers; round numbers either through front-end estimation or to the nearest 100 or 10.

Materials and Resources:

Lesson 1

- Jar of beans – 200+ jelly beans or other dried beans in a clear container
- Sticky notes or cards for student estimates
- Scoop or cup
- Betcha by Stuart J. Murphy
- Teacher Resource 1 - See Appendix A
- Student Resources 1-2 - See Appendix B

Lesson 2

- Number line on the board
- Teacher Resources 3-4– See Appendix A
- Student Resources 3-7– See Appendix B
- Two overhead transparencies

Lesson 3

- Number line on the board
- Teacher Resources 5-6– See Appendix A
- Student Resources 8-11– See Appendix B
- Two overhead transparencies

Development/Procedures:

Lesson 1

Teacher Preparation

- Fill a jar with hundreds of beans or jellybeans. It is not necessary to make an exact count of the beans!
- Pre-read Betcha book, including questions at the end.

Pre-Assessment – How Many Jelly Beans?

- Display a large jar with hundreds of jellybeans or any dry beans. Invite students to examine the jar to determine about how many beans are inside. Students can record answers on sticky notes.
- Share **what** students answered, noting the range of answers.
- Discuss **how** students decided on their answers.
- Distinguish between estimating and guessing. (*Estimates involve strategies.*)
- Key Question: ***Is it okay to make an estimate?*** Discuss - Are we looking for an estimate or an exact number? Is it important?
- Informally assess students' use of strategies.

Launch – Betcha by Stuart Murphy

- Read the book to the class.
- While reading, discuss the various situations where estimates are used in the story, and how the characters decided their estimates.
- Periodically stop and ask questions, such as “What would *you* do?” or “How would you estimate?”
- Discuss actual situations where students might already estimate. *Examples might include ordering pizza for a class, counting how many people are at a crowded restaurant, or taking turns on the computer (measuring time).*

Teacher Facilitation

- Key Question: ***Is it always okay to make an estimate?*** Ask when an estimate would be appropriate or not appropriate. Examples might include determining *how many parents are coming to Back to School Night (appropriate); measuring medicine to give to a child (not appropriate).*
- Impress that there are many more situations when estimates are appropriate.
- Elicit student responses to the question: What are the benefits of estimating? (Possible answers might be: it saves time; it's not always possible to find an exact answer without a calculator or paper-and-pencil; it is sometimes impossible or impractical to make an exact count.)

Student Application

- Model decision-making. Describe a situation that students are familiar with, such as teachers requesting busses to take students on a field trip. Orally, discuss whether or not it would be appropriate to make an estimate

for how many busses are needed. Students may have arguments for either an estimate or an exact count. Encourage students to explain their answers and engage in dialogue amongst pairs, table groups, or whole class.

- Divide students in groups of 3 or 4. Hand each group a situation card (Student Resource - 1 – Situation Cards).
- Give students about 2 minutes to discuss whether they should find an estimate or exact answer for their situation.
- Circulate and observe student discussions. Use checklist for informal monitoring (Teacher Resource 1 – Estimation Observation Checklist).
- Give groups new situation cards to discuss after 2 minutes.
- Regroup the class. As a whole group, discuss various cards.

Closure:

- Revisit the jellybean jar. If we wanted to estimate rather than guess how many beans are inside, we would have to employ a strategy. Review the “layering” strategy explained in the Betcha book. What could be another strategy? Elicit student ideas.
- Demonstrate one of the students’ responses, or model this one: Take one scoop (scoopful or cupful) of jelly beans out. This is called a **referent**. Allow students to count the removed beans. How many more *scoops* or *cupfuls* of beans remain in the jar? What’s a reasonable estimate for the amount in the entire jar? How did obtaining a referent help us?
- Remind students that a good estimate is one that is reasonable and relatively easy to compute. The “best” estimate is not necessarily the one that is closest to the exact answer!

Embedded Assessment

- Student Resource 2 – Exit Ticket
- Answer key is on Teacher Resource 1.

Reteaching/Extension

- Reteaching –Revisit scenarios in Betcha or the real world that children are familiar with (situation cards can help), and encourage children to realize that they estimate often! Help them to recognize that they employ strategies to estimate.
- Extension – Students can explore how they might use referents to help them estimate how many inches of snow fell outside, how many paperclips are in the supply box, or how many ants are crawling across the sidewalk.

Lesson 2

Teacher Preparation

- Cut apart addition expressions used in launch (Student Resource 3 – Addition Expressions).
- Cut apart digit cards for game, and place in paper bags (Student Resource 4 - Digit Cards).
- Make overhead transparencies of the “On Target Sums” directions page, and “On Target Sums - Player A” game board, page 1 (Student Resource Sheet 5 – On Target Sums Game).
- Display a vertical number line on the board with chalk, string, or paper.

Pre-assessment

- Observe students as they place their estimates on the number line described in “Launch,” below.

Launch

- Create a vertical number line on the board with the benchmarks of 0, 100, and 200.
- Give each student a card showing an expression (e.g. $49 + 15$) to place on the number line as they enter the class. Ask them to **estimate** where it should be placed (Student Resource 3 – Addition Expressions).
- Require students to use mental math to estimate their expressions. Do not allow paper and pencil. Allow students to confer with each other if they choose.

Teacher Facilitation

- Discuss the estimates and the number line with the entire class. Possible prompts include:
 - “Was this activity challenging for you? Discuss with a partner.”
 - “Do you agree with the placement of all the cards?”
 - “Who placed card ‘ $16 + 39$ ’? How did you know where to put it? Please share your thinking.”
- Review with students what they learned yesterday about estimates. They are not guesses. They employ strategies. They are often used in the real world because they can save time. Often, exact answers are not necessary.
- Ask students whether they used exact calculation or estimation to place their expression on the number line. Ask students who say that they used estimation to explain their strategies.
- **Review estimation strategies** including front end and rounding. You might also discuss “friendly”, “nice,” or compatible numbers (e.g. 5s, 10, 100s) that are easy to work with but don’t always follow strict rounding rules. For example, $26 + 78$ is more easily seen as $25 + 75 = 100$, rather than $30 + 80 = 110$

- Discuss the concept of a **range** in estimation. Say: How far can you be from the actual answer and still be correct? You might take the example of expression card “ $11 + 44$.” Model the most accurate estimation by rounding the addends ($10 + 40$) and adding. The estimated answer could be 50. What if someone used friendly numbers by changing 11 to 10 and 44 to 50? The estimated answer would be 60. Is that still reasonable? (yes) What if someone rounded both numbers to the nearest 10—11 to 0, and 44 to 0. Is that sum reasonable? (no). We can determine reasonableness by comparing the estimated sum to the exact answer. Here, the actual answer is 55. Estimated sums between 45 and 65 might be reasonable. If we came up with an estimated sum of larger than 65 or less than 45 we might have taken too many liberties in our rounding. Our estimate is not reasonable. Compare this to the jellybean jar from yesterday. What would have been an acceptable range of answers? The point is not to get the exact count, but a reasonable estimate.

Student Application

- Students practice estimating the sums of 2 and 3 digit addends through the “On Target” game (Student Resource Sheet 5 – On Target Sums Game).
- Give each pair of students one bag of digit cards, 6 game pages and 2 pencils. Read the directions aloud to the students, and model 1 round of play on the overhead.
- Allow students to play for about 20 minutes, monitoring all pairs as they work.
- Regroup students as a whole-class, and discuss the mathematics involved in their game. Ask the following questions to facilitate student discussion:
 - “Was this a game of luck or strategy?”
 - What math vocabulary did you use while playing this game?”
 - “How would this game have been different if you were playing for actual sums, and not estimated sums?”
- **Time permitting**, give students Student Resource 6 – Evaluating Estimates to complete. This assignment requires the students to apply what they have practiced regarding estimating sums, and evaluating for reasonableness. There are 3 forms of the assignment, including one for extension, and one with scaffolding for reteaching. If class time does not allow this, assign each student one form of Student Resource 6 – Evaluating Estimates as homework. Answer key is on Teacher Resource 3.

Embedded Assessment

- Use the student checklist to record how students are using rounding to estimate sums, whether they are developing reasonable estimates, and how well they are able to explain their strategies. Teacher Resource Sheet 1 – Estimation Observation Checklist.

- Use Student Resource 7 – Estimating Sums Exit Card. Answer key is on Teacher Resource 4.

Reteaching/Extension

- Scaffold students through solving (estimating) addition word problems by first rounding, then finding the difference of the rounded numbers, then evaluating for reasonableness. In a small reteaching group, explicitly work through each step. This is modeled on Student Resource 6. Display a word bank with terms such as “addend, sum, estimate, actual answer, rounded, strategy” and refer to it. If necessary, work with smaller numbers (say, $34 + 17 + 14$) and/or allow students to use a calculator after they have rounded their numbers.
- Extend students’ understanding by asking them to find the sums of numbers in the 1000s, or by asking them to compare and contrast estimates obtained when applying different strategies (e.g., front-end estimation, rounding to the nearest 10, and compatible numbers) to the same addition problem (e.g. $277 + 7,029$). Is one estimation strategy more accurate than others?
- Consider reading and discussing other literature on estimation including A Million Fish...More or Less by McKissack; How Much is a Million? by David Schwartz.

Lesson 3

Teacher Preparation

- Cut apart subtraction expressions used in launch (Student Resource 8 – Subtraction Expressions).
- Cut apart digit cards and place them in paper bags (Student Resource 4 – Digit Cards).
- Make overhead transparencies of the “On Target Differences” directions page, and “On Target Differences - Player A” game board, page 1 (Student Resource 9 – On Target Differences Game).

Pre-assessment

- Observe students as they place their estimates on the number line described in “Launch,” below.

Launch

- Create a vertical number line on the board with the benchmarks of 0, 50, and 100.

- Give each student a card showing an expression (e.g. $96 - 41$) to place on the number line as they enter the class. They have to **estimate** where it should be placed (Student Resource 8 – Subtraction Expressions).
- Require students to use mental math to estimate their expressions. Do not allow paper and pencil. Do allow students to confer with each other if they want to. Do encourage students to apply the strategies that they learned yesterday regarding rounding or working with friendly numbers.

Teacher Facilitation

- Discuss the estimates and the number line with the entire class. Possible prompts include: “Was it possible or easy for you to subtract these numbers in your head? Why or why not?”
- Ask students whether they used exact calculation or estimation to place their expression on the number line. Ask students who say that they used exact calculation to explain their strategies. Attempt to re-direct them toward estimation strategies, convincing them that estimation saves time and is easier.
- Review with students what they learned over the past 2 days about estimates: they are not guesses. They employ strategies. They are often used in the real world because they can save time. Often, exact answers are not necessary.
- **Review estimation strategies** including front end and rounding. You might also discuss “friendly”, “nice,” or compatible numbers (e.g. 5s, 10, 100s) that are easy to work with but don’t always follow strict rounding rules. For example, $106 - 44$ is more easily seen as $100 - 50 = 50$, rather than $110 - 40 = 70$.
- Discuss the concept of a **range** in estimation. How far can you be from the actual answer, and still be reasonable?

Student Application

- Students practice estimating the differences through the On Target Differences Game (Student Resource 9 – On Target Differences Game).
- Give each pair of students one bag of digit cards (1-7), 6 game pages and 2 pencils. Students must estimate differences, and the first student whose difference falls below the target of 50 is the winner.
- Allow students to play for about 15 minutes, monitoring all pairs as they work.
- Regroup students as a whole-class, and discuss the mathematics involved in their game. Ask the following questions to facilitate discussion:
 - “How did today’s target game compare to the game we played yesterday?”
 - “Without looking back at your paper, tell me the process that you used to estimate differences.”

- “Would this game have been more challenging if you were playing for actual differences, and not estimated differences?”
- Time permitting, give students Student Resource 10 - Estimating Sums and Differences Independent Practice to complete. This page requires that students apply what they have practiced regarding estimating sums and differences. If time does not allow this work in class, assign Student Resource 10 – Estimating Sums and Differences Independent Practice as homework. Answer key is on Teacher Resource 5.

Embedded Assessment

- Continue to use the student checklist, Teacher Resource 1 – Estimation Observation Checklist, to record how students are using rounding to estimate differences, whether they are developing reasonable estimates, and how well they are able to explain their strategies.

Reteaching/Extension

- Scaffold students through solving (estimating) subtraction word problems by first rounding, then finding the difference of the rounded numbers, and finally evaluating for reasonableness. Employing a word bank with terms such as “difference, estimate, actual answer, rounded, strategy” could help. If necessary, work with smaller numbers ($26 - 12$) and/or allow students to use a calculator after they have rounded their numbers.
- Extend students’ understanding by asking them to find the difference of numbers in the 1000s, or by asking them to compare and contrast estimates obtained when applying different strategies (e.g., front-end estimation, rounding to the nearest 10, and compatible numbers) to the same subtraction problem (e.g. $1356 - 475$). Is one estimation strategy more accurate than others?

Summative Assessment

Students will demonstrate an understanding of estimating sums and differences by completing brief constructed response items. The assessment, Student Resource Sheet 11 – Summative Assessment, will integrate the skills that students have been taught so far in this unit, and will require them to use mathematical reasoning and apply mathematics vocabulary to justify their thinking. Answer key is on Teacher Resource 6.

Authors:

Anita Chan
Kemp Mill Elementary
Montgomery County, MD

Elizabeth Sadqi
Pine Crest Elementary
Montgomery County, MD

Appendix A: Teacher Resources

Teacher Resource 1

Estimation Observation Checklist

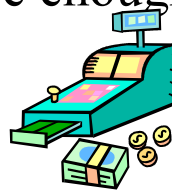
[illegible]

Situation Cards

It's your birthday and your mom wants to make cupcakes for your party. She needs to know how many cupcakes to make.



You want to buy new shoes and a shirt and you only have \$25. Before you get to the checkout, you need to decide if you have enough money.



Third grade is ordering pizza for a party. The teachers need to know how many pizzas to order.



You and your friend decide to take turns on the swing so that each of you get the same amount of time.



You have a bag of popcorn and your best friend wants to share it with you. You want to give her half.

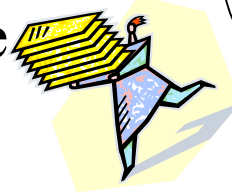


The teachers are organizing their classrooms. They need to know how many student desks to keep in their classrooms.



Exit Ticket

Exact Count or Estimate



Directions: Decide whether to give an estimate or an exact answer.

The art teacher has a stack of construction paper left over in her closet. She wonders how many red, purple, and blue sheets she has. Should she make an exact count or an estimate of how much construction paper she has?

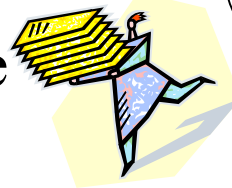
Circle One:

Exact Count

Estimate

Explain your thinking.

Exact Count or Estimate



Directions: Decide whether to give an estimate or an exact answer.

The art teacher has a stack of construction paper left over in her closet. She wonders how many red, purple, and blue sheets she has. Should she make an exact count or an estimate of how much construction paper she has?

Circle One:

Exact Count

Estimate

Explain your thinking.

Exit Ticket

Answer Key

Exact Count or Estimate



Directions: Decide whether to give an estimate or an exact answer.

The art teacher has a stack of construction paper left over in her closet. She wonders how many red, purple, and blue sheets she has. Should she make an exact count or an estimate of how much construction paper she has?

Circle One:

Exact Count

Estimate

Explain your thinking.

Answer may be exact or estimate. Students should justify how they made their decision.
For example, Exact – The teacher needs to know if there is enough paper for the whole class. Estimate – It would take too long to count all of the papers.

Addition Expressions

$23 + 61$	$43 + 4$
$49 + 15$	$75 + 19$
$18 + 82$	$11 + 44$
$59 + 3$	$91 + 2$
$37 + 12$	$48 + 25$
$53 + 31$	$44 + 4$
$8 + 76$	$24 + 45$
$17 + 34$	$57 + 35$
$16 + 39$	$22 + 37$
$88 + 14$	$28 + 23$

$106 + 39$	$172 + 13$
$136 + 38$	$155 + 23$
$98 + 48$	$84 + 79$
$14 + 127$	$52 + 109$
$75 + 65$	$174 + 22$

Digit Cards

1	2	3	4
5	6	7	

1	2	3	4
5	6	7	

On Target Sums Game

Topic: Estimating Sums

Object: Be the first to reach the target sum of 300, and win!

Groups: Pairs

Materials:

- Each player must have an On Target Sums game board, and pencil
- Each pair shares a set of Digit Cards (1-7) in a brown paper bag

Directions:

1. Player with Game Board A starts.

Player A:

- Pick two digit cards from the bag.
- Arrange them to create a 2-digit number.
For example, if you picked a 1 and a 7, you can make 17 or 71.
- Write your number below the starting 25 on your game board.
- Round the two addends, and write your rounded numbers \rightarrow to the right of the actual numbers.
- Add to find your estimated sum. Ask Player B to confirm that your estimate is reasonable.
- Follow the arrow. Recopy your estimated sum in the boxes on the left.
- This is the end of your turn. If you have reached the 300-point target, you win!

Player B:

- Pick two digit cards from the bag.
- Arrange them to create a 2-digit number.
For example, if you picked a 1 and a 7, you can make 17 or 71.
- Write your number below the starting 23 on your game board.
- Round the two addends, and write your rounded numbers \rightarrow to the right of the actual numbers.
- Add to find your estimated sum. Ask Player A to confirm that your estimate is reasonable.
- Follow the arrow. Recopy your estimated sum in the boxes on the left.
- This is the end of your turn. If you have reached the 300-point target, you win!

2. Continue taking turns. The first player who reaches 300 is the winner!

On Target Sums – Player A

2

5

→

→

+

→

→

+

Does the order of your digits matter?

→

→

+

Copy this number on the next page to continue playing.

Copy your number on the previous page to continue playing.

<input type="text"/>	<input type="text"/>	<input type="text"/>	→	<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>	→		<input type="text"/>	<input type="text"/>
				+		
				<hr/>		
	<input type="text"/>	<input type="text"/>		<input type="text"/>	<input type="text"/>	<input type="text"/>

<input type="text"/>	<input type="text"/>	<input type="text"/>	→	<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>	→		<input type="text"/>	<input type="text"/>
				+		
				<hr/>		
	<input type="text"/>	<input type="text"/>		<input type="text"/>	<input type="text"/>	<input type="text"/>

Are your estimates reasonable?

<input type="text"/>	<input type="text"/>	<input type="text"/>	→	<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>	→		<input type="text"/>	<input type="text"/>
				+		
				<hr/>		
	<input type="text"/>	<input type="text"/>		<input type="text"/>	<input type="text"/>	<input type="text"/>

Copy this number on the next page to continue playing.

Source 5 (cont.)

Copy your number on the previous page to continue playing.

<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>



<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>

+

<input type="text"/>	<input type="text"/>	<input type="text"/>
----------------------	----------------------	----------------------



<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>



<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>

+

<input type="text"/>	<input type="text"/>	<input type="text"/>
----------------------	----------------------	----------------------

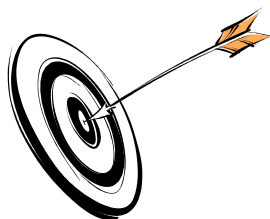
<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>



<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>

+

<input type="text"/>	<input type="text"/>	<input type="text"/>
----------------------	----------------------	----------------------



On Target Sums – Player B

2	3	→		
		→	+	

			→			
			→	+		

Does the order of your digits matter?

			→			
			→	+		

Copy this number on the next page to continue playing.			→			
--	--	--	---	--	--	--

Copy your number on the previous page to continue playing.

Student Resource 5 (cont.)

<div><div></div><div></div><div></div></div>	→	<div><div></div><div></div><div></div></div>
<div><div></div><div></div></div>	→	<div><div></div><div></div></div>
		+

<div><div></div><div></div><div></div></div>	→	<div><div></div><div></div><div></div></div>
<div><div></div><div></div></div>	→	<div><div></div><div></div></div>
		+

Are your estimates reasonable?

<div><div></div><div></div><div></div></div>	→	<div><div></div><div></div><div></div></div>
<div><div></div><div></div></div>	→	<div><div></div><div></div></div>
		+

Copy this number on the next page to continue playing.

Copy your number on the previous page to continue playing.

Student Resource 5 (cont.)



+



--	--	--





+



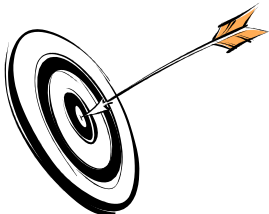
--	--	--



+



--	--	--



Evaluating Estimates

Independent Work



Students borrow books from the media center. Last week:

44 students borrowed
animal books

22 students borrowed
joke books

19 students borrowed
biographies

The Media Specialist estimated that 100 students borrowed books last week. Is her estimate reasonable or not reasonable?

Circle one: Yes, the estimate is reasonable. No, the estimate is not reasonable.

Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Evaluating Estimates

Independent Work

Answer Key



Students borrow books from the media center. Last week:

44 students borrowed
animal books

22 students borrowed
joke books

19 students borrowed
biographies

The Media Specialist estimated that 100 students borrowed books last week. Is her estimate reasonable or not reasonable?

Circle one: Yes, the estimate is reasonable.

No, the estimate is not reasonable.

Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Answers will vary. Explanation demonstrates that 100 is not in an acceptable range and is therefore not reasonable. Students may show:

$$\begin{array}{r}
 44 \rightarrow 40 \\
 22 \rightarrow 20 \\
 + \quad 19 \rightarrow 20 \\
 \hline
 80
 \end{array}$$

An estimate of 80 would be more reasonable.

Evaluating Estimates

Extension



Students borrow books from the media center. Last week:

44 students borrowed
animal books

22 students borrowed
joke books

19 students borrowed
biographies

The Media Specialist estimated that 100 students borrowed books last week. Is her estimate reasonable or not reasonable?

Circle one: Yes, the estimate is reasonable. No, the estimate is not reasonable.

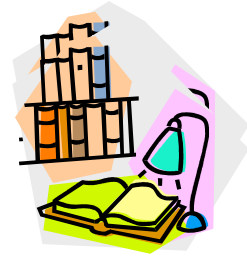
Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Some students also borrowed poetry books. How many students might have borrowed poetry books so that the Media Specialists' estimate of 100 students would be reasonable? Use words, numbers or pictures in your explanation.

Evaluating Estimates

Extension

Answer Key



Students borrow books from the media center. Last week:

44 students borrowed
animal books

22 students borrowed
joke books

19 students borrowed
biographies

The Media Specialist estimated that 100 students borrowed books last week. Is her estimate reasonable or not reasonable?

Circle one: Yes, the estimate is reasonable.

No, the estimate is not reasonable.

Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Answers will vary. Explanation demonstrates that 100 is not in an acceptable range and is therefore not reasonable. Students may show:

$$\begin{array}{r}
 44 \rightarrow 40 \\
 22 \rightarrow 20 \\
 + 19 \rightarrow 20 \\
 \hline
 80
 \end{array}
 \quad
 \begin{array}{l}
 \text{An estimate} \\
 \text{of 80 would} \\
 \text{be more} \\
 \text{reasonable.}
 \end{array}$$

Some students also borrowed poetry books. How many students might have borrowed poetry books so that the Media Specialists' estimate of 100 students would be reasonable? Use words, numbers or pictures in your explanation.

Answers will vary. Explanation demonstrates that a number that rounds to 20 needs to be added. Students may show:

$$\begin{array}{r}
 44 \rightarrow 40 \\
 22 \rightarrow 20 \\
 19 \rightarrow 20 \\
 + (\geq 15) \rightarrow 20 \\
 \hline
 100
 \end{array}$$

Evaluating Estimates

Reteaching



Students borrow books from the media center. Last week:

44 students borrowed
animal books

22 students borrowed
joke books

19 students borrowed
biographies

- a) First, round each number.
- b) Next, find the sum of the rounded numbers.

	Exact number	Rounded number
Animal Books	44	
Joke Books	22	
Biographies	19	
		Sum:

- c) Compare your estimate to the Media Specialist's estimate of 100.
- d) Decide whether the Media Specialist's estimate was reasonable or not.

Circle one: Yes, the estimate is reasonable. No, the estimate is not reasonable.

- e) Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Evaluating Estimates

Reteaching

Answer Key



Students borrow books from the media center. Last week:

44 students borrowed
animal books

22 students borrowed
joke books

19 students borrowed
biographies

- First, round each number.
- Next, find the sum of the rounded numbers.

	Exact number	Rounded number
Animal Books	44	40
Joke Books	22	20
Biographies	19	20
		Sum: 80

- Compare your estimate to the Media Specialist's estimate of 100.
- Decide whether the Media Specialist's estimate was reasonable or not.

Circle one: Yes, the estimate is reasonable.

No, the estimate is not reasonable.

- Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Answers will vary. Explanation demonstrates that 100 is not in an acceptable range and is therefore not reasonable. Students may show:

$$\begin{array}{r}
 44 \quad 40 \\
 22 \quad 20 \\
 + \quad 19 \quad 20 \\
 \hline
 \quad \quad 80
 \end{array}$$

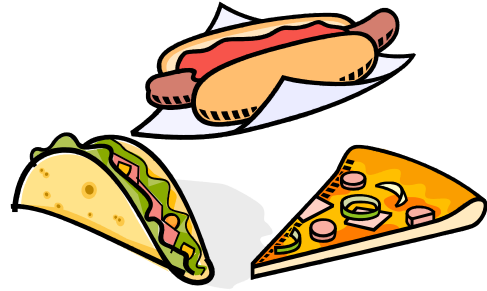
An estimate of 80 would be more reasonable.

Estimating Sums Exit Card

Name: _____ Date: _____

The cafeteria served three different lunches today.

77 students bought hot dogs.
215 students bought tacos.
355 students bought pizza.



About how many students bought lunch today? _____

What strategy did you use to estimate? Explain your strategy using words and/or numbers.

--

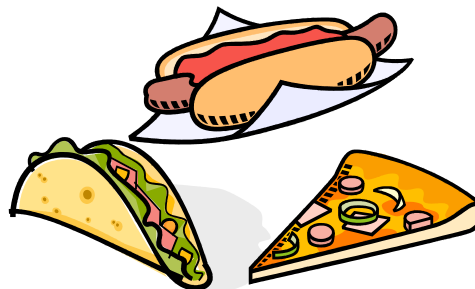
Estimating Sums Exit Card

Answer Key

Name: _____ Date: _____

The cafeteria served three different lunches today.

77 students bought hot dogs.
215 students bought tacos.
355 students bought pizza.



About how many students bought lunch today? 570 - 700

What strategy did you use to estimate? Explain your strategy using words and/or numbers.

Answers will vary. Explanation demonstrates that students used some type of rounding to estimate the sum. Student may show:

$$\begin{array}{r} 77 \longrightarrow 70 \\ 215 \longrightarrow 200 \\ + 355 \longrightarrow 300 \\ \hline 570 \end{array}$$

(Front End)

$$\begin{array}{r} 77 \longrightarrow 100 \\ 215 \longrightarrow 200 \\ + 355 \longrightarrow 400 \\ \hline 700 \end{array}$$

(Nearest 100)

$$\begin{array}{r} 77 \quad 80 \\ 215 \quad 220 \\ + 355 \quad 360 \\ \hline 660 \end{array}$$

(Nearest 10)

Subtraction Expressions

$96 - 41$	$106 - 44$
$52 - 12$	$38 - 14$
$67 - 28$	$187 - 96$
$43 - 7$	$63 - 36$
$89 - 14$	$100 - 72$
$125 - 41$	$98 - 45$
$83 - 46$	$62 - 57$
$91 - 83$	$48 - 32$
$58 - 33$	$78 - 27$
$143 - 54$	$21 - 3$

On Target Differences Game

Topic: Estimating Differences

Object: Be the first to reach the target difference of 50, and win!

Groups: Pairs


Materials:

- Each player must have a On Target Differences game board, and pencil
- Each pair shares a set of Digit Cards (1-7) in a brown paper bag


Directions:

1. Player with Game Board A starts.

Player A:

- Pick two digit cards from the bag.
- Arrange them to create a 2-digit number.
For example, if you picked a 1 and a 7, you can make 17 or 71.
- Write your number below the starting 525 on your game board.
- Round the two numbers, and write your rounded numbers  to the right of the actual numbers.
- Subtract to find your estimated difference. Ask Player B to confirm that your estimate is reasonable.
- Follow the arrow. Recopy your estimated difference in the boxes on the left.
- This is the end of your turn. If you have reached the 50-point target, you win!

Player B:

- Pick two digit cards from the bag.
- Arrange them to create a 2-digit number.
For example, if you picked a 1 and a 7, you can make 17 or 71.
- Write your number below the starting 523 on your game board.
- Round the two numbers, and write your rounded numbers  to the right of the actual numbers.
- Subtract to find your estimated difference. Ask Player A to confirm that your estimate is reasonable.
- Follow the arrow. Recopy your estimated difference in the boxes on the left.
- This is the end of your turn. If you have reached the 50-point target, you win!

2. Continue taking turns. The first player who reaches 50 is the winner!

On Target Differences – Player A

5	2	5	→			
			→	-		

--	--	--

			→			
			→	-		

--	--	--

Does the order of your digits matter?

			→			
			→	-		

--	--	--

Copy this number on the next page to continue playing.

Source 9 (cont.)

Copy your number on the previous page to continue playing.

			→			
			→			
				-		
				<hr/>		

			→			
			→			
				-		
				<hr/>		

Are your estimates reasonable?

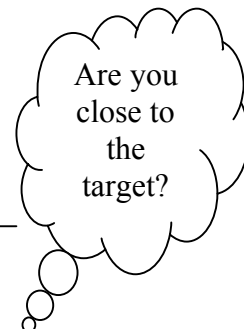
			→			
			→			
				-		
				<hr/>		

Copy this number on the next page to continue playing.

ac rce 9 (cont.)

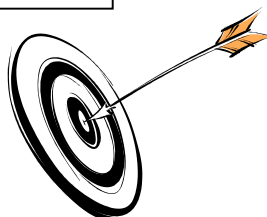
Copy your number on the previous page to continue playing.

			→			
			→			
				-		



			→			
			→			
				-		

			→			
			→			
				-		



On Target Differences – Player B

5	2	3	→			
			→	-		

--	--	--

			→			
			→	-		

--	--	--

Does the order of your digits matter?

			→			
			→	-		

--	--	--

Copy this number on the next page to continue playing.

Copy your number on the previous page to continue playing.

			→			
			→			
				-		

			→			
			→			
				-		

Are your estimates reasonable?

			→			
			→			
				-		

Copy this number on the next page to continue playing.

ac rce 9 (cont.)

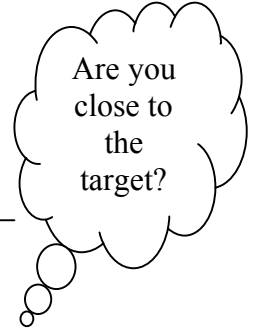
Copy your number on the previous page to continue playing.

<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>



<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>

-



<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>



<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>

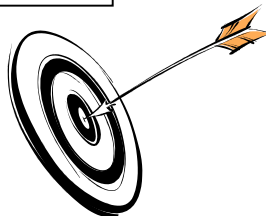
-

<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>



<input type="text"/>	<input type="text"/>	<input type="text"/>
	<input type="text"/>	<input type="text"/>

-



Estimating Sums and Differences

Independent Practice

Brian counted the animals he saw in the park.

He counted 176 birds.
He counted 12 chipmunks.
He counted 64 squirrels.



About how many animals did Brian count in all? _____

Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

About how many **more** squirrels than chipmunks did he count? _____

Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

About how many **more** birds than squirrels did he count? _____

Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Estimating Sums and Differences

Independent Practice

Answer Key

Brian counted the animals he saw in the park.

He counted 176 birds.
He counted 12 chipmunks.
He counted 64 squirrels.



About how many animals did Brian count in all? 250

Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Answers will vary. Explanation demonstrates that students rounded and then used their rounding to estimate their sum.

$$\begin{array}{r} 176 \longrightarrow 180 \\ 12 \longrightarrow 10 \\ + 64 \longrightarrow 60 \\ \hline 250 \end{array}$$

About how many **more** squirrels than chipmunks did he count? 50

Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Answers will vary. Explanation demonstrates that students rounded and then used their rounding to estimate their difference.

$$\begin{array}{r} 64 \longrightarrow 60 \\ - 12 \longrightarrow 10 \\ \hline 50 \end{array}$$

About how many **more** birds than squirrels did he count? _____

Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Answers will vary. Explanation demonstrates that students rounded and then used their rounding to estimate their difference.

$$\begin{array}{r} 176 \longrightarrow 180 \\ - 64 \longrightarrow 60 \\ \hline 120 \end{array}$$

Summative Assessment

Hilltop Elementary School students enjoyed a music concert.

135 third graders watched the concert.
41 fourth graders watched the concert.
272 fifth graders watched the concert.



About how many more 5th graders than 3rd graders watched the concert? Record your estimate below.

a) Estimate _____

b) Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

c) Amy said that the total number of third, fourth, and fifth grade students watching the concert is about 440. Is this a reasonable estimate?

Circle one: Yes, the estimate is reasonable. No, the estimate is not reasonable.

d) Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Summative Assessment – *Answer Key*

Hilltop Elementary School students enjoyed a music concert.

135 third graders watched the concert.
41 fourth graders watched the concert.
272 fifth graders watched the concert.



About how many more 5th graders than 3rd graders watched the concert? Record your estimate below.

- a) Estimate 100 - 200
- b) Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Answers will vary. Explanation demonstrates that students used some type of rounding to estimate the difference. Student may show:

$$\begin{array}{r} 272 \longrightarrow 200 \\ - 135 \longrightarrow 100 \\ \hline 100 \end{array}$$

(Front End)

$$\begin{array}{r} 272 \longrightarrow 300 \\ - 135 \longrightarrow 100 \\ \hline 200 \end{array}$$

(Nearest 100)

$$\begin{array}{r} 272 \longrightarrow 270 \\ - 135 \longrightarrow 140 \\ \hline 130 \end{array}$$

(Nearest 10)

- c) Amy said that the total number of third, fourth, and fifth grade students watching the concert is about 440. Is this a reasonable estimate?

Circle one: Yes, the estimate is reasonable. No, the estimate is not reasonable.

- d) Use what you know about estimation to explain your thinking. Use words and/or numbers in your explanation.

Explanations will vary. Each should demonstrate application of an estimation strategy. Students may mention that there is a range of appropriate answers. Student may show:

$$\begin{array}{r} 135 \quad 100 \\ 41 \quad 40 \\ + 272 \quad 200 \\ \hline 340 \end{array}$$

(Front End)

$$\begin{array}{r} 135 \quad 100 \\ 41 \quad 0 \\ + 272 \quad 300 \\ \hline 400 \end{array}$$

(Nearest 100)

$$\begin{array}{r} 135 \quad 140 \\ 41 \quad 40 \\ + 272 \quad 270 \\ \hline 450 \end{array}$$

(Nearest 10)